

Autoresonant control of the many-electron dynamics in nonparabolic quantum wells

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The optical response of nonparabolic quantum wells is dominated by a strong peak at the plasmon frequency. When the electrons reach the anharmonic regions, resonant absorption becomes inefficient. This limitation is overcome by using a chirped laser pulse in the autoresonant regime. By direct simulations using the Wigner phase-space approach, the authors prove that, with a sequence of just a few pulses, electrons can be efficiently detrapped from a nonparabolic well. For an array of multiple quantum wells, they can create and control an electronic current by suitably applying an autoresonant laser pulse and a slowly varying dc electric field. © 2007 American Institute of Physics. [DOI: 10.1063/1.2761246]

Small semiconductor devices, such as quantum dots and quantum wells, have attracted considerable attention in recent years, particularly for possible applications in the emerging field of quantum computing.¹ For quantum devices working with many electrons,² it is crucial to understand the properties of the self-consistent electron dynamics and its response to external electric fields. Particular attention has been devoted to intersubband transitions in semiconductor quantum wells, which take place on the meV energy scale and involve excitation frequencies of the order of the terahertz.³ Several theoretical and computational studies have investigated the electron response, mainly using Hartree-Fock semiconductor Bloch equations⁴ or density functional theory.⁵ For perfectly parabolic confinement, the electron response is dominated by the Kohn mode,^{6,7} consisting of rigid oscillations of the electron gas at the effective plasmon frequency. For nonparabolic confinement, the Kohn mode still dominates the initial response. However, when the electrons reach the anharmonic regions, the resonance condition is lost and absorption becomes inefficient.

In this letter, we show that this limitation can be overcome by resorting to *autoresonant* excitation.^{8–10} Basically, autoresonant excitation occurs when a classical nonlinear oscillator is excited by an oscillating force with slowly varying frequency: $F(t) = \epsilon \cos[\omega_0(t-t_0) + \frac{1}{2}\alpha(t-t_0)^2]$, where ϵ is the excitation amplitude and ω_0 the frequency, which matches the linearized oscillator frequency; α is the rate of variation of the excitation frequency. For $|\alpha| \ll \omega_0^2$ and ϵ above a certain threshold, the instantaneous oscillator frequency becomes “locked” to the instantaneous excitation frequency so that the resonance condition is always satisfied. In that case, the amplitude of the oscillations grows indefinitely and without saturation, until of course some other effect kicks in. For the single-particle case the threshold behaves as $\epsilon_{th} \sim |\alpha|^{3/4}$.⁸

In order to study the self-consistent electron dynamics, we make use of the Wigner representation of quantum mechanics. A mixed quantum state is represented by a function of the phase space variables plus time: $f(x, v, t)$ (we deal with one-dimensional problems), which evolves according to the Wigner equation,

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{im_*}{2\pi\hbar} \int \int d\lambda dv' e^{im_*(v-v')\lambda} f(x, v', t) \times \left[V\left(x + \frac{\lambda\hbar}{2}, t\right) - V\left(x - \frac{\lambda\hbar}{2}, t\right) \right] = \left(\frac{\partial f}{\partial t} \right)_{\text{scatt}}, \quad (1)$$

where m_* is the effective electron mass and $V(x, t)$ is the total potential acting on the electrons. The latter is composed of three terms: (i) the confining potential $V_{\text{conf}}(x)$, (ii) the autoresonant oscillating potential $V_{\text{auto}} = xF(t)$, and (iii) the Hartree potential $V_H(x, t)$, which obeys Poisson's equation $V_H'' = (e^2/\epsilon) \int_{-\infty}^{\infty} f dv$, where e is the absolute electron charge and ϵ is the effective dielectric constant. We consider wide quantum wells (≈ 100 nm) at moderate electron temperatures ($\approx 2T_F$), for which the exchange and correlation corrections can be neglected.^{11,12}

The right-hand side of Eq. (1) models disorder or phonon scattering in the form of a friction-diffusion term,¹³

$$\left(\frac{\partial f}{\partial t} \right)_{\text{scatt}} = 2\gamma \frac{\partial(vf)}{\partial v} + D_v \frac{\partial^2 f}{\partial v^2} + D_x \frac{\partial^2 f}{\partial x^2}, \quad (2)$$

where γ is the relaxation rate (inverse of the relaxation time T_1), and D_v and D_x are diffusion coefficients in velocity and real space, respectively, which are related to the decoherence time T_2 . In order for Eq. (1) to preserve the positivity of the density matrix associated to the Wigner distribution function, the scattering term must be in Lindblad form.¹³ This is automatically achieved¹⁴ if the above coefficients respect the inequality $D_v D_x \geq \gamma^2 \hbar^2 / 4m_*^2$.

We focus on confining potentials that can be approximated by a parabola at the bottom of the well: $V_{\text{conf}}(x) \approx \frac{1}{2}\omega_0^2 m_* x^2 + \dots$, where the frequency ω_0 can be related to a fictitious homogeneous positive charge of density n_0 via the relation $\omega_0^2 = e^2 n_0 / m_*$. We then normalize time to ω_0^{-1} , space to the harmonic oscillator length $L_{\text{ho}} = \sqrt{\hbar / m_* \omega_0}$, velocity to $\sqrt{\hbar \omega_0 / m_*}$, energy to $\hbar \omega_0$, and the electron density to n_0 .

As initial condition, we take a Maxwell-Boltzmann velocity distribution with temperature $T_e > T_F$ and a Gaussian density profile with peak density n_e . This is not an exact stationary state, but it evolves very little if no perturbation is applied. (The precise form of the initial state is irrelevant for our purposes, provided it is localized at the bottom of the

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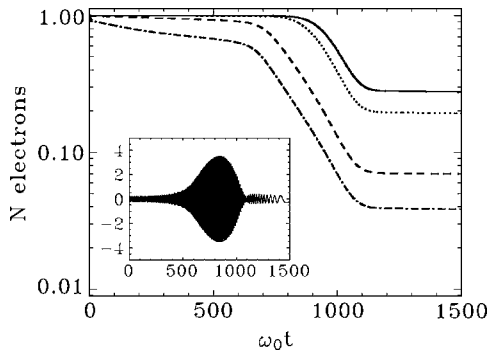


FIG. 1. Single Gaussian well. Total number of electrons in the well, normalized to its value at $t=0$, for different values of the filling fraction: $\eta=0$ (solid line), $\eta=0.1$ (dotted), $\eta=0.5$ (dashed), and $\eta=1$ (dot dash). Inset: Time evolution of the dipole for $\eta=0.5$.

well.) We define the filling fraction as $\eta=n_e/n_0 \leq 1$: the limit case $\eta=0$ corresponds to very dilute densities, for which the Hartree potential is negligible.

We use typical parameters for semiconductor quantum wells:⁵ effective electron mass and dielectric constant $m^*=0.067m_e$ and $\epsilon=13\epsilon_0$; volume density $n_0=5 \times 10^{16} \text{ cm}^{-3}$, $\omega_0=1.35 \times 10^{13} \text{ s}^{-1}$, $\hbar\omega_0=8.9 \text{ meV}$, and $L_{\text{ho}}=11.3 \text{ nm}$. For $\eta=1$, this yields a maximum surface density for the electrons $n_s=1.35 \times 10^{11} \text{ cm}^{-2}$ and a maximum Fermi temperature $T_F=85.7 \text{ K}$. The electron temperature is taken to be $T_e=2\hbar\omega_0 \approx 200 \text{ K}$. The relaxation time is $T_1 \approx 70 \text{ ps}$, which corresponds to a relaxation rate $\gamma/\omega_0=0.001$. The diffusion coefficient in velocity space is $D_v=\gamma\sqrt{k_B T_e}/m^*$.

First, we consider a single quantum well with a Gaussian potential: $V_{\text{conf}}(x)=-V_0 \exp(-x^2/2\sigma^2)$, with $V_0=\sigma^2 m^* \omega_0^2$, $\sigma=4L_{\text{ho}} \approx 45 \text{ nm}$, and an overall width of the quantum well of $L=24L_{\text{ho}} \approx 270 \text{ nm}$. This is in line with recent experiments on wide parabolic quantum wells, which can reach a width of several hundred nanometers.¹²

The electron gas is excited with a chirped pulse, with $\alpha=-0.001\omega_0^2$, $t_0=500\omega_0^{-1}$, and amplitude $\epsilon=0.2 > \epsilon_{\text{th}}$, in normalized units. The latter corresponds to an electric field of the order of 0.1 mV/nm , which can be easily achieved experimentally. Chirped pulses were also suggested to excite transitions in two-level quantum systems¹⁵—a situation quite different from the wide quantum wells considered here, where many levels are present and progressively excited by means of the autoresonant technique.

Figure 1 shows the dipole of the electron distribution (average of x) and the total number of electrons in the well [integral of $f(x,v,t)$ in the phase space] normalized to its initial value, for different values of the filling fraction η . The dipole starts increasing rapidly at $t=t_0$, i.e., when the time-dependent frequency of the laser field crosses the linear resonance ω_0 . At $\omega_0 t \approx 900$, the electrons that have been accelerated by the laser field are finally ejected from the well, leaving behind a remnant population that is still sitting at the bottom of the well. The dipole then decreases rapidly and remains close to zero thereafter. The number of electrons in the well is reduced by one order of magnitude at the end of the run. The slight initial decrease observed for large electron densities ($\eta=1$) represents a small evaporation due to Coulomb repulsion.

By applying a sequence of four identical pulses, it is possible to reduce the number of electrons by almost three

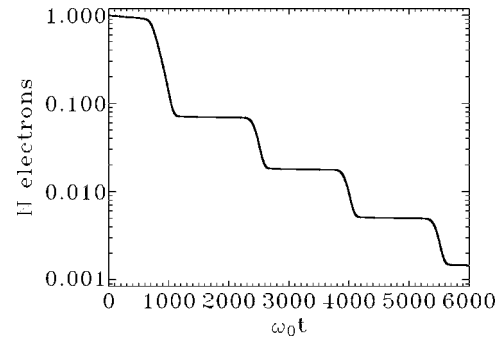


FIG. 2. Time evolution of the total number of electrons for a series of four laser pulses; $\eta=0.5$.

orders of magnitude, as shown in Fig. 2. We stress that, for a nonchirped pulse ($\alpha=0$) or for a chirped pulse below the autoresonant threshold ($\epsilon < \epsilon_{\text{th}}$), virtually no electrons leave the well. In addition, the effect is still observed when the laser frequency is mismatched with respect to the harmonic oscillator frequency ω_0 . Even for a mismatch of $\pm 10\%$, the same number of electrons are ejected from the well.

As a second application, we consider a periodic array of quantum wells, with a cosinusoidal confining potential $V_{\text{conf}}=-V_0 \cos(2\pi x/\lambda)$, where λ is the width of each well, and $V_0=m^* \omega_0^2 \lambda^2/2\pi$. Such periodic superlattices can be practically realized as multilayer semiconductor heterostructures¹⁶ and there have been recent attempts at simulating these structures using Bose-Einstein condensates trapped in optical lattices.¹⁷ Here, we neglect the Hartree potential and consider noninteracting electrons (this amounts to assuming $\eta \leq 1$). We take $\gamma=0.002\omega_0$, $\alpha=-0.002\omega_0^2$, $\omega_0 t_0=300$, and $\epsilon=0.2\hbar\omega_0/L_{\text{ho}}$. Initially, each quantum well is occupied by a single electron, represented by a minimum uncertainty packet. We use a periodic computational box with spatial period equal to 3λ .

The idea is to create an electron current by applying a suitable laser pulse. From the previous study, we have learned that the electrons can be efficiently extracted from the well. By applying a small constant (dc) electric field, we can force the electrons to always leave the well from one side and thus create an electric current. For this, we use a small dc field ($E_0=0.1\hbar\omega_0/eL_{\text{ho}} \approx 0.08 \text{ mV/nm}$) to polarize the array of quantum wells.¹⁸ In Fig. 3, we plot the evolution of the average velocity of the electron distribution $\langle v \rangle = \int f v dv dx / \int f dv dx$.

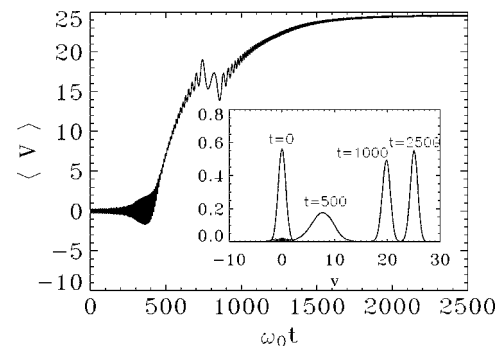


FIG. 3. Evolution of the average velocity for an array of quantum wells with dc bias. Inset: velocity distribution of the electron population at $\omega_0 t=0, 500, 1000$, and 2500 . Velocity is measured in units of $\sqrt{\hbar\omega_0}/m^*$.

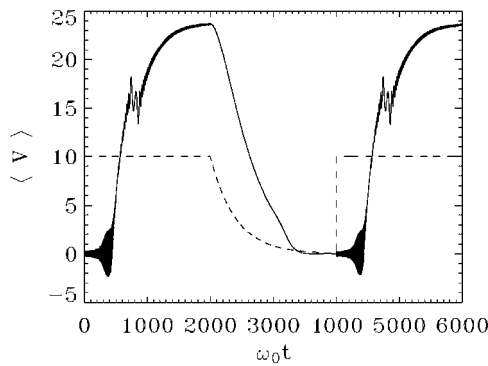


FIG. 4. Evolution of the average velocity for an array of quantum wells with dc bias, in the case of two laser pulses. The dashed line represents the evolution of the dc field, in arbitrary units.

At $t=t_0$, a positive current starts building up and, by the end of the run, the average velocity reaches a value of $\langle v \rangle \approx 25$ in our normalized units. This value corresponds to the asymptotic velocity reached by an electron subjected to a constant electric field $E_0=0.1$ in the presence of a friction coefficient $2\gamma=0.004$, i.e., $v_{\max}=E_0/2\gamma=25$ (normalized units). This shows that all electrons have been detrapped from the potential well and accelerated by the dc field. The electron velocity distribution (Fig. 3, inset) confirms this scenario.

In order to make the whole process reversible, one would need to retrap the electrons inside the potential well. This can be achieved in the following way (Fig. 4): (i) after the current has been generated, the oscillating pulse is switched off suddenly at $\omega_0 t=2000$; (ii) at the same time, the dc electric field E_0 is switched off adiabatically between $\omega_0 t=2000$ and $\omega_0 t=4000$ (exponential decrease with time constant $\tau=400\omega_0^{-1}$): during this phase, all electrons are re-trapped and the current goes back to zero; (iii) at $\omega_0 t=4000$, the dc electric field is switched on again and another autoresonant laser pulse (identical to the first one) is used to excite the current once again. This procedure can be repeated several times, so that the electric current can be

switched on and off, and can even change sign if one changes the sign of the dc field.

In summary, we presented numerical evidence that the electron gas in wide nonparabolic quantum wells can be efficiently excited with an autoresonant laser pulse. Nonlinear effects were triggered using a relatively weak pulse, even when the laser frequency is poorly tuned. These techniques could be used to achieve better control of the electron dynamics in quantum solid-state devices.

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